If retro occurs at $t_{min} < \tau < t_{max}$ the postretro cost is given by $f(\tau - t_{min}) = \frac{1}{3}a_T(\tau - t_{min})$ and the optimum divert point is $S(\tau) = \frac{1}{2}a_T\tau^2 - \frac{1}{3}\frac{1}{2}a_T(\tau - t_{min})^2$. The solution for the total cost is then the solution of

$$\tau^{1/2}F = \int (2/3)\tau^{1/2}a_T\,d\tau \tag{22}$$

The boundary condition which determines the solution is the total cost for the case of $\tau = t_{min}$ in which case there is no postretro cost and the total engagement cost for $t_{min} < \tau < t_{max}$ is

$$F(\tau) = (4/9)a_T\tau - (1/9)a_T t_{min}(t_{min}/\tau)^{1/2} \tag{23}$$

The optimal solutions for the preretro aim point, the postretro aim point and the total cost (total interceptor velocity required) of the engagement are summarized in Table 1.

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Simple Waves in One-Dimensional Unsteady Nonequilibrium Dissociative Gasdynamics

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I. Introduction

STUDY of simple waves in general relaxation hydrodynamics has been the subject of investigation by Coburn.[1-3] Using the set of equations given by Stupochenko and Stakhanov,[4] Coburn has arrived at a number of interesting theorems. As the study has been quite general some of the theorems escaped physical interpretations. Accordingly, we have in this Note, made an attempt to give physical interpretation to one of the important theorems due to Coburn by specializing it to the case of ideal dissociative gasdynamics.

II. Simple Waves in One-Dimensional Unsteady Motion

Following Coburn,[2] the characteristic manifolds for the system of equations given in Ref. 2, can be expressed as

$$X = I\hat{C}^2 - J \tag{1}$$

where

$$I = (Fk + GT)K + 2\bar{q}_1 F \tag{2a}$$

$$J = (FkC_\infty^2 + GTC_0^2)K + 2\bar{q}_1 FC_\infty^2 \tag{2b}$$

$$\hat{C}^2 = 1/\phi(\phi_0 + v^j\phi_j)^2, \quad L = \phi_0 + v^j\phi_j \tag{2c}\dagger$$

The bicharacteristics of the same system are determined by

$$dx^j/d\sigma = \tfrac{1}{2}\partial X/\partial\phi_j = ILv^j - J\phi_j \tag{3}$$

$$dx^0/d\sigma = \tfrac{1}{2}\partial X/\partial\phi_0 = IL = \pm(IJ\phi)^{1/2} \tag{4}$$

Equations (3) and (4) yield

$$dx^j/dx^0 = v^j + \hat{C}n^j \tag{5}$$

where $\hat{C}$ is given by Eq. (1) and n^j, the space-unitized normal to the characteristic space-time manifolds S_3, is given by

$$n^j = \phi^j/\phi^{1/2} \tag{6}$$

If α, β be the parameters along the two families of bicharacteristics then, for the one-dimensional unsteady case, Eq. (5) can further be written as

$$\partial x/\partial\alpha = (u + \hat{C})(\partial t/\partial\alpha), \quad \partial x/\partial\beta = (u - \hat{C})(\partial t/\partial\beta) \tag{7}$$

For the theory of simple waves in the x, t plane we use the following definition: a family of simple waves consists of a family of bicharacteristics (with parameter, β = variable or α = constant, along each curve of the family) such that

$$\partial q/\partial\beta = \partial S/\partial\beta = \partial\rho/\partial\beta = 0 \tag{8a}$$

$$\partial v_j/\partial\beta = 0 \tag{8b}$$

Next, we consider those consequences of energy equations, the definition of rate equation[2] and the geometry of bicharacteristics Eqs. (1) and (7) and a relation of the type

$$\partial\alpha/\partial t = (\hat{C} - u)/2\hat{C}(\partial t/\partial\alpha)^{-1}, \quad \partial\alpha/\partial x = 1/2\hat{C}(\partial t/\partial\alpha)^{-1} \tag{9}$$

which are valid in both the aforementioned cases. The energy and rate equations in terms of the derivatives of S and q with respect to α can be expressed as

$$KT(dS/d\alpha) = 2\bar{q}^2(\partial t/\partial\alpha) \tag{10}$$

$$dq/d\alpha = 2\bar{q}(\partial t/\partial\alpha) \tag{11}$$

and elimination of $\partial t/\partial\alpha$ from Eqs. (10) and (11) yields

$$KT(dS/d\alpha) = \bar{q}(dq/d\alpha) \tag{12}$$

Further substituting the value of $\bar{q}/K = -(\partial e/\partial q)$, the Eq. (12) can be written as

$$de/dq = -T(dS/dq) \tag{13}$$

This fact can be stated in the form of the following theorem.

Theorem 1

If the rate of change of internal energy with respect to the relaxation variable is positive (negative) then as one moves from one simple wave to another, entropy decreases (increases) as relaxation variable increases.

Now, using the chain rule for differentiation, the definition Eq. (8), the expressions for $\partial\alpha/\partial t$, $\partial\alpha/\partial x$ from Eq. (9), the values of A, B, C as given in Ref. 1, and Eq. (12), the equation of continuity and motion for simple waves under consideration can be written as

$$\hat{C}(d\rho/d\alpha) + \rho(du/d\alpha) = 0 \tag{14}$$

$$\rho\hat{C}(du/d\alpha) + A\rho(d\rho/d\alpha) + ((B\bar{q}/KT) + C)dq/d\alpha = 0 \tag{15}$$

As in the equilibrium gasdynamics, Eq. (14) can be interpreted as follows.

Theorem 2

In both cases, as one moves from one simple wave to another, the density increases (decreases) as particle speed decreases (increases).

Further eliminating $d\rho/d\alpha$ from Eqs. (14) and (15) and using Eq. (12) we get

$$du/dq = -\hat{C}/(\rho(\hat{C}^2 - A))(B(dS/dq) + C) \tag{16}$$

The result Eq. (16) has been interpreted by Coburn[2] in the form of the following.

Theorem 3

If the rate of increase of internal energy or entropy with respect to the relaxation variable is such that $[C + B(dS/dq)]$ is positive then: 1) in the supersonic case $\hat{C}^2 > A$, the particle speed, u, is decreasing as the relaxation variable increases; and 2) in the subsonic case $\hat{C}^2 < A$, the particle speed, u is increasing as the relaxation variable increases.

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† Symbols used in Eq. (2) have the same meaning as in Ref. 2.

What exactly is meant by the positiveness of the expression $[C + B(dS/dq)]$ can be made clear by specializing the preceding theorem to the case of an ideal dissociating gas as introduced by Lighthill.[5]

For an ideal dissociating gas, we have, using Capiaux and Washington,[6] (under usual notations)

$$A = (\partial p/\partial \rho)S, \alpha* = a_f^2 = RT\{[(1 + \alpha*)]^2/3 + (1 + \alpha*)\}$$

$$B = (\partial p/\partial S)\rho, \alpha* = (\rho T/3)(1 + \alpha*)$$

$$C = (\partial p/\partial \alpha*)\rho, S = (\rho/3)(1 + \alpha*)(\mu_1 - \mu_2) + \rho\{RT - (D/3)(1 + \alpha*)\}$$

where μ_1 and μ_2 are the chemical potentials of the atoms and molecules, respectively and $\alpha*$ is the mass-fraction.

Further from the second law of thermodynamics we write

$$TdS = dh - (1/\rho)dp - (\mu_1 - \mu_2)d\alpha*$$

Substituting the values of A, B, C and $dS/d\alpha*$ from the preceding Eq. (16), we get

$$du/d\alpha* = -\{\hat{C}/[3(\hat{C}^2 - a_f^2)\rho]\}(3RT - D\alpha* - D) \qquad (17)$$

Conclusion

In the terminology of Capiaux and Washington[6] the first term in the bracket of Eq. (17), i.e. $3RT$ corresponds to a "sensible" internal energy e_S, and the second term $(D\alpha*)$ is taken as e_C, the internal energy of chemical bonding. Keeping this terminology in view we state the following theorem on simple waves in ideal dissociating gasdynamics: if the ratio of the difference of sensible internal energy e_S and the chemical bond energy e_C to the dissociation energy D per unit mass is such that it exceeds unity then the particle speed u, decreases or increases as the dissociation variable increases, according as the flow is supersonic or subsonic.

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Calculated Pulsating One-Dimensional Detonations with Induction-Zone Kinetics

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Nomenclature

a = sound speed
C = specific constant—pressure heat capacity
D = detonation velocity
$E\dagger$ = activation energy
E^* = reaction rate parameter
f = D^2/D_{CJ}^2, degree of overdrive
k = rate multiplier
p = pressure
Q = heat release parameter
r = reaction rate
R = specific gas constant
x = distance
t = time
T = absolute temperature
v = specific volume
ε = reaction rate parameter
γ = isentropic exponent
λ = degree of reaction
ρ = density

Subscripts
0 = initial state
1, 2 = species 1 and 2

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ONE-DIMENSIONAL pulsating detonations similar to those previously reported[1] for first-order Arrhenius kinetics have been calculated for more realistic kinetics: a simple mockup of the branching-chain mechanism characterizing a large class of gaseous fuel/oxygen systems. The previous work used an idealized model of a gaseous detonation system consisting of a mixture of two polytropic gases with the same heat capacity and molecular weight, one transformed into the other by a first-order Arrhenius reaction. It was inspired by a linear analysis of the hydrodynamic stability of the one-dimensional steady solution in the piston-supported (overdriven) regime. This analysis[2] showed that the steady solution is, for some values of the parameters, unstable to one-dimensional perturbations as well as to three-dimensional ones; hence the interest in the simpler one-dimensional time-dependent calculation. This calculation gave in the unstable case a strongly pulsating detonation, with peak-to-peak front pressure amplitude not much less than the pressure of the unstable steady solution.

The computer program[3] for the one-dimensional time-dependent calculation considers a single chemical reaction and assumes constant heat capacities. Our mockup of the branching-chain mechanism is constructed within this constraint. An accurate description requires much more detail: perhaps ten or more elementary chemical reactions and a comparable number of chemical species with temperature-dependent heat capacity. A computer program for calculating the one-dimensional steady solution with such a realistic representation is available at this laboratory; some results obtained with it are presented below for comparison.

The equation of state and rate used in the present time-dependent calculation are

$$pv = T, \quad E = (C - 1)T - \lambda Q; \quad C = C_1 + \lambda(C_2 - C_1)$$
$$r = k\rho(\lambda + \varepsilon)(1 - \lambda)e^{-E\dagger/T - E^*\lambda} \qquad (1)$$

The system is a mixture of two species with constant heat capacities C_1 and C_2. Other symbols are the same as in Ref. 1, with the exception of the dimensionless constants ε and E^* here introduced into the rate. As before, p, v, and T are in units of their values in the initial state, E and Q are in units of RT_0, and C is in units of R. The unit of time is the time required for a particle in the steady solution to react to $\lambda = \frac{1}{2}$ after it is shocked. The distance unit is a_0 times the time unit. The half-reaction-zone length is the distance of the $\lambda = \frac{1}{2}$ point from the steady shock.

All of the calculations reported here are started from an approximate steady-solution configuration, generated by programming the piston to follow the velocity history of a particle passing through the desired steady solution, specified by the final constant piston velocity approached in this prescription. This final piston velocity or the corresponding steady detonation velocity D is the single problem parameter characterizing the boundary conditions. We measure it by the "degree of over-